

Activity 3: Chaos in a simple system: Part II

In this activity, we study a very simple model of population dynamics. This model is known as the *logistic map*. Although the model is exceptionally simple, its behavior can be unpredictable. And yet, despite the unpredictability of this and other chaotic systems, we'll see in this activity that there is some kind of order hidden in the chaotic dynamics.

To make our activity concrete, we'll use a contrived example of mosquitos on an isolated island. We'll explore a simple mathematical model that mimics the seasonal variation of the mosquito population.

This activity continues on where the last one left off.

1. Make sure that you thoroughly understand each part of the last activity. If you go on without understanding what you've done, you won't understand this activity.
2. Using the program `logistic_map`, what happens to the mosquito population if $r = 3.5$? (Recall that r is the reproduction rate of the mosquitoes and must be between 0 and 4.)
3. Sketch, by hand a graph of the x_∞ vs. r , for values $0 \leq r \leq 3.5$, where x_∞ is "final" value of x_i (i.e., the value after $i \rightarrow \infty$). If it helps, use the program `logistic_map` to play around with different values of r and find x_∞ .
4. Run the program `rvsx.m`, which is a program to do the same thing you sketched by hand in the previous part. Does it look like your sketch? Do you understand what it means? How come the plot has a *split*? The splitting is called *bifurcation*.
5. Run the program `logistic_map.m` again but use values $3.54 < r < 3.9$. Try a whole bunch of different values. What conclusions can you draw about the mosquito population for these reproduction rates? Is this realistic? Explain.
6. Modify the program `rvsx.m` to make a bifurcation plot for the values $3 < r < 3.9$. Woah! What is going on? How does this plot relate to our discussion of the Sierpinski gasket in Activity 1? Explain.
7. What are the implications of all of this? That is, what is the big picture here? What does this activity tell us about the physical world?