Homework 13:

Do problem 7.1 in the text

Solution:

**Part (a)** It didn’t come from the air—it came from within the strawberries. Introducing the sugar (dissolving in the water on the surface of the berries) caused an osmotic pressure difference. The resulting osmotic flow pulled water out of the strawberries to the surface.

**Part (b)**

The picture of knowledge being forced, i.e. "permeation", into our heads (by ourselves or by others) seems like a better fit than it diffusing there randomly, i.e. osmosis.

Do problem 7.2 in the text.

Solution:

**Part (a)**

<table>
<thead>
<tr>
<th>Concentration</th>
<th>Pressure (mmHg)</th>
<th>Concentration (moles/m³)</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>535</td>
<td>$1.75 \times 10^{25}$</td>
<td>$7.2 \times 10^4$</td>
</tr>
<tr>
<td>2</td>
<td>1016</td>
<td>$3.5 \times 10^{25}$</td>
<td>$13.6 \times 10^4$</td>
</tr>
<tr>
<td>2.74</td>
<td>1518</td>
<td>$4.8 \times 10^{25}$</td>
<td>$20.4 \times 10^4$</td>
</tr>
<tr>
<td>4</td>
<td>2082</td>
<td>$7.0 \times 10^{25}$</td>
<td>$28 \times 10^4$</td>
</tr>
<tr>
<td>6</td>
<td>3072</td>
<td>$10.5 \times 10^{25}$</td>
<td>$41 \times 10^4$</td>
</tr>
</tbody>
</table>

\[
C = \frac{1 \text{ g sugar}}{100 \text{ g } H_2O} \times \frac{1 \text{ g } H_2O}{10^{-6} \text{ m}^3} \times \frac{1 \text{ mole}}{342 \text{ g sugar}} = 1.75 \times 10^{25} \text{ m}^{-3}
\]

\[
p = 535 \text{ mmHg} \times \frac{1.01 \times 10^5 \text{ Pa}}{752 \text{ mmHg}} = 7.2 \times 10^4 \text{ J/m}^3(\text{ N/m}^2)
\]
data = {{1.75, 7.2}, {3.5, 13.6}, {4.8, 20.4}, {7.0, 28}, {10.5, 41}};
line = Fit[data, {1, x}, x]
Show[Plot[line, {x, 0, 12}, PlotStyle -> Red], ListPlot[data, Joined -> True,
  PlotMarkers -> Automatic, PlotStyle -> PointSize[0.3], PlotRange -> {{0, 12}, {0, 41}}]]

0.696539 + 3.87359 x
The slope of the resulting graph is $3.87 \times 10^{-21} \text{ J} = \frac{\text{pressure}}{\text{concentration}}$.

The van’t Hoff relation would say: $p = c k_B T \Rightarrow \frac{p}{c} = k_B T \Rightarrow 4.1 \times 10^{-21} \text{ J}$

**Part (b)**

<table>
<thead>
<tr>
<th>$T(\degree \text{C})$</th>
<th>Pressure (mmHg)</th>
<th>$T(\text{K})$</th>
<th>Pressure (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>505</td>
<td>280</td>
<td>$6.78 \times 10^4$</td>
</tr>
<tr>
<td>14</td>
<td>525</td>
<td>287</td>
<td>$7.05 \times 10^4$</td>
</tr>
<tr>
<td>22</td>
<td>548</td>
<td>295</td>
<td>$7.36 \times 10^4$</td>
</tr>
<tr>
<td>32</td>
<td>544</td>
<td>305</td>
<td>$7.31 \times 10^4$</td>
</tr>
<tr>
<td>36</td>
<td>567</td>
<td>309</td>
<td>$7.62 \times 10^4$</td>
</tr>
</tbody>
</table>

These data lie on a line with a slope of $\approx (247/\text{K})T$

Take pressure at $15^\circ \text{C}$ from part (a), $p = 1.75 \times 10^{25} \text{ m}^{-3}k_B T_r$

and the pressure at $15^\circ \text{C}$ from part (b), $p = (247 \text{ J/K})T_r$.

Equate them and get $k_B = 1.38 \times 10^{-23} \text{ J/K}$ or $k_B T_r = 4.1 \times 10^{-21}$ as expected.

Do problem 7.5 in the text.

Solution:

**Part (a)** At separation of $2R = 2(10 \text{ nm}) = 20 \text{ nm}$ depletion forces (i.e. "pressure") from the globular proteins will crowd the objects together.

**Part (b)** The concentration is $c = \frac{\phi}{V_{mol}} = \frac{0.3}{\frac{4}{3} \pi (10 \text{ nm})^3} = 7.2 \times 10^{22} \text{ m}^{-3}$

This is about 0.12 mm. The osmotic pressure (van’t Hoff) is

$p = c k_B T_r = (4.1 \times 10^{21} \text{ J})(7.2 \times 10^{22} \text{ m}^{-3}) \approx 290 \text{ Pa}$

Thus the pressure in the depletion zone is small compared to atmospheric pressure ($10^5 \text{ Pa}$).

The decrease in free energy by putting the surfaces together is:
\[ \Delta F = (k_B T_r) (\Delta V) c = (290 \text{ J/m}^3)(2 \times 10 \text{ nm} \times 10 \text{ \mu m}^2) = 5.7 \times 10^{-17} \text{ J}, \]

which is much bigger than \( k_B T_r \). So, the surfaces will definitely stick together.