Homework 08:

Do problem 5.4 in the text

Solution:

**Part (a)**

Recall Newton’s second law

\[ F = ma = \frac{dv}{dt}m \]

The only force is friction so

\[ m\frac{dv}{dt} = F_{\text{frict}} = v_{\text{diff}}\zeta = -v(6\pi\eta r) \]

(for a spherical bacteria), with a negative direction due to the friction opposing the motion.

\[ m\frac{dv}{dt} = (-6\pi\eta r)v \implies v(t) \sim e^{-t/\tau} \text{ where } \tau = \frac{m}{6\pi\eta r} \]

So the solution must be:

\[ v(t) = v_0e^{-t/(m/6\pi\eta r)} \]

which makes

\[ \tau = \frac{m}{6\pi\eta r} = \frac{\rho \frac{4}{3}\pi r^3}{6\pi\eta r} = \frac{2(1000 \text{ kg/m}^3)(10^{-6} \text{ m})^3}{9(9 \times 10^{-4} \text{ N/m}^2 \cdot \text{s})(10^{-6} \text{ m})} = 2.47 \times 10^{-7} \text{ sec} \]

To get the distance, just integrate velocity over time.

\[ \int_0^\infty v(t) \, dt = \int_0^\infty v_0 e^{-t/\tau} \, dt = -v_0\tau [e^{-\infty/\tau} - e^{-0/\tau}] = v_0 t \]

\[ x = (10^{-6} \text{ m/s})(2.47 \times 10^{-7} \text{ s}) = 2.5 \times 10^{-13} \text{ m} \]

→ The bacteria doesn’t coast very far.

**Part (b)**

As long as our random walk has \( \Delta t > \tau \) then the steps should be uncorrelated.
Do problem 5.5 in the text

Solution:

**Part (a)**

From Eqns 5.17 and 5.18

\[
P = \frac{8L\eta Q}{\pi R^4} = \frac{8(0.1 \text{ nm})(\sim 10^{-3} \text{ N/m}^2 \text{ s})(500 \times 10^{-6} \text{ m}^3)}{\pi \left( \frac{0.025 \text{ m}}{2} \right)^4} = 5.2 \text{ Pa}
\]

5.2 Pa is a small fraction of the $10^5$ Pa for atmospheric pressure.

**Part (b)**

Each second $500 \times 10^{-6} \text{ m}^3$ of blood flows through the aorta. The pressure required is 5.2 Pa. Each second, how much work is done? Use dimensional analysis.

\[
W = F \cdot d = (P \times \text{Area}) \times \left( \frac{\text{Volume}}{\text{Area}} \right) = \text{Pressure} \times \text{Volume}
\]

\[
= (5.2 \text{ N/m}^2)(500 \times 10^{-6} \text{ m}^3) = 2.6 \times 10^{-3} \text{ Watts}
\]

This is a *lot* less than the 100 Watts for basic metabolic rate.

**Part (c)**

Just recall Eq. (5.17) and how we came up with it.

\[
v(r) = \frac{(R^2 - r^2)}{4L\eta} P
\]