Exam 3 Solutions

(16 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer. **Fill in your answers on the bubble sheet.**

1.1. The sound of a trumpet playing a note is qualitatively different than the sound of a flute playing a note at the same pitch. Why is that?
- a. The two notes have different amplitudes
- b. The two notes have different durations
- c. The two notes have different fundamental frequencies
- d. The two notes have different phases
- e. The two notes have different strengths of harmonics

1.2. Without using a multiple lens/mirror setup, a converging lens can produce a virtual image.
- a. True
- b. False

f is positive

p must be positive

$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1}$

q could be negative, if $\frac{1}{p} > \frac{1}{f}$

1.3. Without using a multiple lens/mirror setup, a diverging lens can produce a real image.
- a. True
- b. False

f is negative

$\frac{1}{f}$ is negative, $\frac{1}{p}$ is positive $\Big\}$ $\frac{1}{f} - \frac{1}{p}$ must be negative

1.4. You compare the diffraction pattern between two single ("wide") slits, one at a time. Slit A produces a pattern that has the maxima and minima more spaced out than the pattern produced by Slit B. Which is true?
- a. Slit A is narrower than Slit B
- b. Slit A is wider than Slit B

wider slit → faster oscillations in diffraction pattern

So slit B must be wider

1.5. Which is more likely to focus closer to a converging lens… a blue laser or a red laser?
- a. Blue laser
- b. Red laser

Blue light = higher index of refraction

→ shorter focal length

1.6. What kind of lenses do far-sighted people need to correct their vision?
- a. Converging
- b. Diverging

to help them bring far away images to the near point (~25 cm)

1.7. In slit problems, when can you make the approximation that the rays come out from the slits parallel to each other?
- a. When the slit size/spacing is much smaller than L
- b. When the slit size/spacing is much larger than L
- c. When the part of the diffraction pattern you are interested in is much smaller than L
- d. When the part of the diffraction pattern you are interested in is much larger than L

if slits are small, rays to point will be parallel

1.8. In slit problems, when can you make the approximation that $\sin\theta = y/L$?
- a. When the slit size/spacing is much smaller than L
- b. When the slit size/spacing is much larger than L
- c. When the part of the diffraction pattern you are interested in is much smaller than L
- d. When the part of the diffraction pattern you are interested in is much larger than L

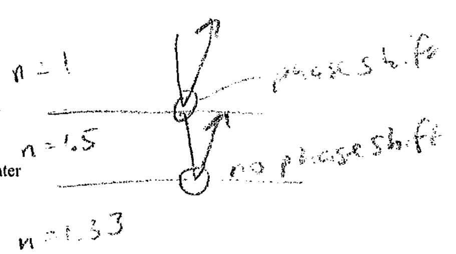
small angle: if $\frac{y}{L}$ = small

then $\sin\theta = \theta = \tan\theta$

y must be << L

1.9. How did Young do his famous "double-slit" experiment?
- a. He used a laser to ensure coherence between the two slits
- b. He used a third slit to produce light that was coherent at the two slits
- c. He used a gas lantern and did not worry about coherence between slits

1.10. A thin layer of oil (thickness t, index of refraction = 1.5) is covering a pool of water (infinitely thick, index of refraction = 1.33). For light shining directly on the pool from above and reflecting back, which of the following is true?
- a. The light will interfere constructively with itself when $2t = m\lambda_{vacuum}$
- b. The light will interfere constructively with itself when $2t = (m + ½)\,\lambda_{vacuum}$
- c. The light will interfere constructively with itself when $2t = m\lambda_{vacuum}/n_{oil}$
- d. The light will interfere constructively with itself when $2t = (m + ½)\,\lambda_{vacuum}/n_{oil}$
- e. The light will interfere constructively with itself when $2t = m\lambda_{vacuum}/n_{water}$
- f. The light will interfere constructively with itself when $2t = (m + ½)\,\lambda_{vacuum}/n_{water}$



Constructive: $2t = \underbrace{(m + \tfrac{1}{2})}_{\text{because of phase shift}} \lambda_{oil}$

$2t = (m + \tfrac{1}{2}) \frac{\lambda}{n_{oil}}$

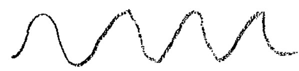
1.11. For diffraction from a single "wide" slit, which is true of the central point (y=0)?

single wide slit : sinc^2 function

a. The intensity at y=0 is infinite
b. The intensity is finite, but typically much larger than the surrounding maxima
c. The intensity is finite, and about the same magnitude as the surrounding maxima
d. The intensity is finite, and substantially smaller than the surrounding maxima
e. The intensity is zero

1.12. For diffraction from two infinitely narrow slits, which is true of the central point (y=0)?

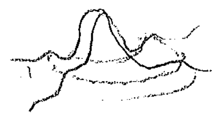
2 slits : $\cos^2$ function



a. The intensity at y=0 is infinite
b. The intensity is finite, but typically much larger than the surrounding maxima
c. The intensity is finite, and about the same magnitude as the surrounding maxima
d. The intensity is finite, and substantially smaller than the surrounding maxima
e. The intensity is zero

1.13. For diffraction from a circular aperture, which is true of the central point (r=0)?

circle aperture : Bessel function^2



a. The intensity at y=0 is infinite
b. The intensity is finite, but typically much larger than the surrounding maxima
c. The intensity is finite, and about the same magnitude as the surrounding maxima
d. The intensity is finite, and substantially smaller than the surrounding maxima
e. The intensity is zero

1.14. Suppose you shine a laser through a piece of foil which has two identical holes shaped like tiny llamas, spaced apart by a distance *d* (you may assume *d* is twice as big as the size of each llama). What will the diffraction pattern look like?

a. The same as a pattern from a single llama
b. Two single-llama patterns, superimposed on one another
c. A single-llama pattern, modulated by a double slit pattern

llama pattern × double slit pattern

1.15. The electric field in a light wave is described by the following equation: $\vec{E} = A\hat{x}\cos\left[k\left(\frac{y+z}{\sqrt{2}}\right) - \omega t\right]$. In what direction is it traveling? (The vectors given as answer choices are all unit vectors.)

a. $\hat{x}$
b. $-\hat{x}$
c. $\frac{\hat{y}+\hat{z}}{\sqrt{2}}$
d. $\frac{-\hat{y}-\hat{z}}{\sqrt{2}}$
e. $\frac{\hat{y}-\hat{z}}{\sqrt{2}}$
f. $\frac{-\hat{y}+\hat{z}}{\sqrt{2}}$

This is $\vec{k}\cdot\vec{r}$
so k-vector is
$k\left(\underbrace{\frac{\hat{y}+\hat{z}}{\sqrt{2}}}_{\to \text{direction}}\right)$

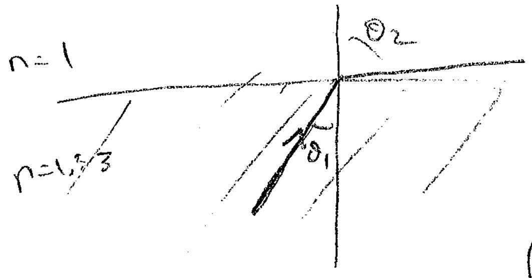
1.16. Same equation. In what direction is the electric field oscillating?

a. From $\hat{x}$ to $-\hat{x}$
b. From $\frac{\hat{y}+\hat{z}}{\sqrt{2}}$ to $\frac{-\hat{y}-\hat{z}}{\sqrt{2}}$
c. From $\frac{\hat{y}-\hat{z}}{\sqrt{2}}$ to $\frac{-\hat{y}+\hat{z}}{\sqrt{2}}$

Amplitude is in $\hat{x}$ direction, so field will oscillate between $+\hat{x}$ (when cosine is positive) and $-\hat{x}$ (when cosine is negative).

(16 pts) **Problem 2.**

(a) If you are underwater, and shine a laser at the surface, at what angles from the perpendicular will none of the laser light emerge from the surface? ($n_{water} = 1.33$; $n_{air} = 1.00$)



Critical angle when $\theta_2 = 90°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

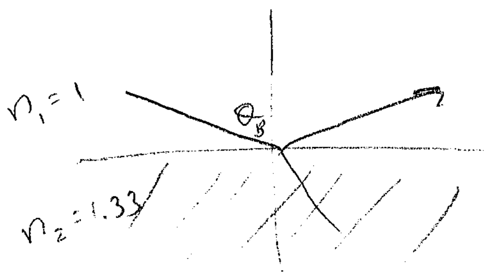
$(1.33)\sin\theta_1 = (1)\sin 90°$

$\theta_1 = 48.8$

$\boxed{\text{For all angles } \theta > 48.8 \text{ T.I.R. will occur}}$

(b) At what angle above the horizon will sunlight reflecting off of a lake be completely polarized to you?

Brewster's angle



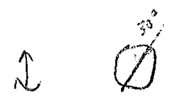
$\tan\theta_B = n_2/n_1$

$\theta_B = \tan^{-1}(1.33/1)$

$\theta_B = 53.1°$ (from perpendicular)

From horizon: $90 - 53.1 = \boxed{36.9°}$

(c) If you shine vertically polarized laser light that at a (perfect) polarizer whose transmission axis is angled at 30° clockwise away from the vertical, (i.e., it would completely allow light through that is polarized at a 30° angle clockwise away from vertical), what fraction of the laser intensity will pass through?



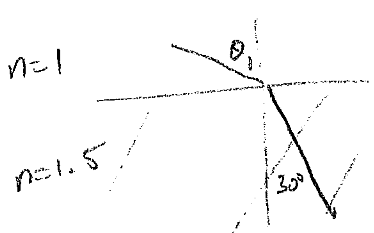
E-field: $\cos 30°$ gets through

$I \propto E^2$

so intensity: $\cos^2 30°$ gets through

$= 75\%$

(d) You want a laser beam going from air into glass to have a 30° angle measured from the perpendicular, inside the glass. At what angle (from the perpendicular) should you set the laser beam at in the air?



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$(1)\sin\theta_1 = (1.5)$

$\boxed{\theta_1 = 48.6°}$

(12 pts) **Problem 3**. Explain the following, in 1-2 sentences each. Do NOT use equations, but you can use pictures if you like.

(a) Why is angular resolution (in radians) of a telescope limited to $1.22\lambda/D$?

The diffraction pattern of a circle  makes every point source of light spread out when it comes through lens. $\theta = \frac{1.22\lambda}{D}$ is the place where the spreading out from two point sources can be distinguished because the peaks are separated far enough 

peak of second at minimum of first.

(b) Why do rainbows form when white light passes through a prism?

Due to dispersion in the glass (different n's for different λ's), the colors (wavelengths) that make up the light all bend at slightly different angles. This causes them to spread out, forming a rainbow.

(c) Why do cameras need lenses? (I.e., what would happen if you tried to make a camera without a lens?)

The lens causes light rays coming from particular spots (e.g. someone's left ear) to meet up again at a particular spot on the film / digital sensor. Without a lens, the film/sensor would just receive an unrecognizable jumble of light rays coming from everywhere at once.

(d) Why is the *angular* magnification used to describe the magnification produced by magnifying glasses and telescopes?

Angular size (θ) governs how big something "looks" to you — how much detail you will be able to make out. Therefore angular magnification, the ratio of angular sizes, is a helpful metric to describe the effect of a lens or telescope.

(12 pts) **Problem 4.** (a) Joanna is near-sighted, and her eyes have a far point of about 100 cm. Assuming she does not have anything else wrong with her eyes (like astigmatism, which we didn't talk about), what should the focal length of her corrective lenses be? Be sure to specify whether f is plus or minus.

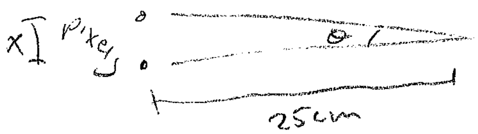
The lenses need to bring things at infinity to 100 cm, where she can see them. Also note the image will be virtual, so $q = -100\,\text{cm}$.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \rightarrow f = \left(\frac{1}{p} + \frac{1}{q}\right)^{-1}$$

$$f = \left(\frac{1}{\infty} + \frac{1}{-100}\right)^{-1}$$

$$\boxed{f = -100\,\text{cm}}$$

(b) TVs and computer monitors are made up of pixels, which are colored dots very closely spaced together. For someone with a near point of 25 cm, how close does the Rayleigh criterion predict that the dots need to be, in order to make viewing individual pixels completely impossible without the aid of a magnifying device? Assume a maximum pupil diameter of 9 mm and a minimum visible wavelength of 390 nm. (The actual pixel spacing needed will not be as tight as your answer predicts, due to smaller average pupil diameters, larger wavelengths for the light emitted by pixels, and the fact that the optically-sensitive molecules in your retina are not infinitely small.)



Rayleigh: $\theta = \frac{1.22\lambda}{D}$

Also, from picture $\theta = \frac{x}{25\,\text{cm}}$

Set equal: $\frac{1.22\lambda}{D} = \frac{x}{25\,\text{cm}}$

$$x = \frac{1.22(390\times10^{-9}\,\text{m})}{(9\times10^{-3}\,\text{m})}(25\times10^{-2}\,\text{m})$$

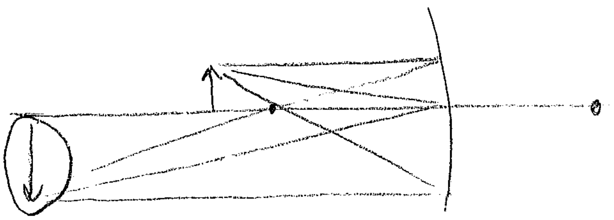
$$\boxed{x = 1.32\times10^{-5}\,\text{m}}$$

$= 13.2\,\mu\text{m}$

That is 76 dots/mm, or 1922 dots/inch

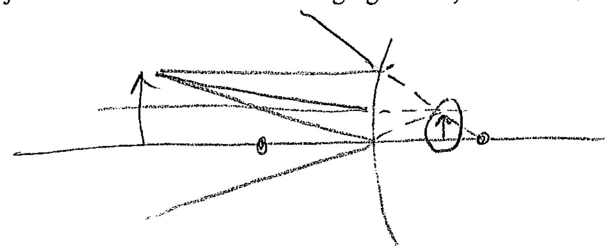
(12 pts) **Problem 5.** Draw accurate ray diagrams for the following situations to indicate where each image will be formed, how large it will be, and whether it will be real or virtual. Use at least three rays for each diagram. Use dashed lines for virtual rays, if present. No equations are necessary, but you are of course free to use equations to double check your final image position if you wish.

(a) An object 20 cm to the left of a converging mirror, f = +15 cm.



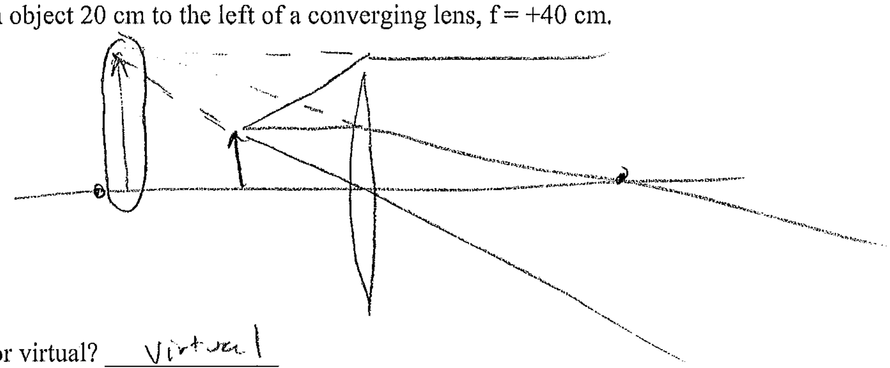
Real or virtual? real

(b) An object 20 cm to the left of a diverging mirror, f = –10 cm.



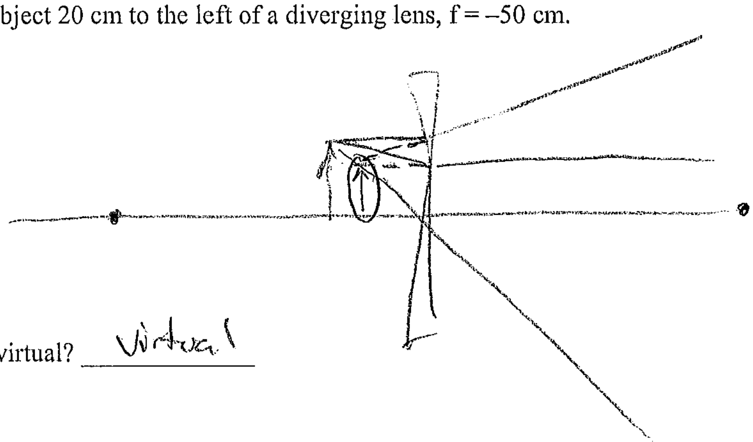
Real or virtual? virtual

(c) An object 20 cm to the left of a converging lens, f = +40 cm.



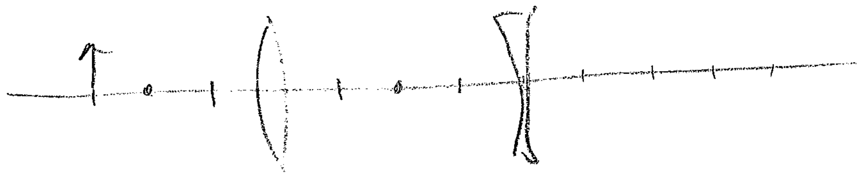
Real or virtual? Virtual

(d) An object 20 cm to the left of a diverging lens, f = –50 cm.



Real or virtual? Virtual

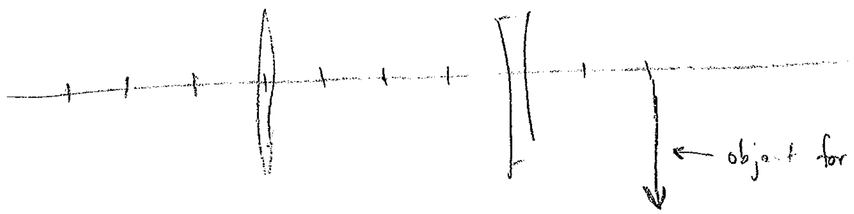
(12 pts) **Problem 6.** An object is placed 30 cm to the left of lens 1(converging, f = +20 cm). Lens 1 is placed 40 cm to the left of lens 2 (diverging, f = –50 cm). Where will the final image be formed? Will it be real or virtual? What will the magnification be? You do not have to provide ray diagrams for this problem, although you are certainly welcome to draw them if that will help you visualize the situation.



Lens 1: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \rightarrow q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1}$

$q = \left(\frac{1}{20} - \frac{1}{30}\right)^{-1} = 60$ cm

$M_1 = \frac{-q}{p} = -\frac{60}{30} = -2$



← object for 2nd lens = image from first lens

$\underline{p = -20\text{cm}}$ ★

$q = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1}$

$= \left(\frac{1}{-50} - \frac{1}{-20}\right)^{-1} = \boxed{+33.3 \text{ cm}}$

$M_2 = \frac{-q}{p} = \frac{-33.33}{-20} = +1.67$

$M_{tot} = M_1 \times M_2 = (-2)(+1.67)$

$= \boxed{-3.33}$

q = +33.3 cm (relative to lens 2)

real vs. virtual: real (since it's on right of lens 2)

M_{tot} = −3.33

(12 pts) **Problem 7.** You set up a diffraction apparatus like the one I used in class, where a 633 nm laser shines on various slits, and forms a diffraction pattern on a screen 10 m away.

(a) If two infinitely narrow slits are used, separated by 100×10^{-6} m, how far apart will the maxima be?

2 slit maxima: $d\sin\theta = m\lambda$

small angle: $\sin\theta \simeq \frac{y}{L}$

$$\frac{dy}{L} = m\lambda$$

$$y = \frac{m\lambda L}{d}$$

separation: $\Delta y = \frac{\lambda L}{d} = \frac{(633\times10^{-9}m)(10m)}{(100\times10^{-6}m)} = \boxed{.0633m}$ → this proves it's small angle, since $y \ll L$

6.33 cm

(b) If a single (wide) slit is used, and the width of the central maxima (as measured by the distance between the first minima on either side) is measured to be 10 cm, what was the width of the slit?

single slit minima: $a\sin\theta = m\lambda$ (except not $m=0$)

small angle: $\sin\theta \simeq y/L$

$$a\,y/L = m\lambda$$

$$y = \frac{m\lambda L}{a}$$

distance between y_{+1} and y_{-1} = $\frac{2\lambda L}{a}$

$$(.10m) = \frac{2(633\times10^{-9}m)(10m)}{a}$$

$$a = \boxed{.0001266m}$$ 126.6 μm

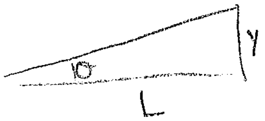
(c) If a diffraction grating is used, with 600 lines per millimeter, how far away will the first maximum be from the center? (I'm counting the maximum at the center as the "zeroeth" maximum.)

$600\frac{\text{lines}}{mm}$ → spacing $d = \frac{1mm}{600} = 1.67\times10^{-6}m$

grating maxima: $d\sin\theta = m\lambda$

$$\sin\theta = \frac{m\lambda}{d}$$

1^{st} maxima: $\theta = \sin^{-1}\left(\frac{(1)(633\times10^{-9}m)}{(1.67\times10^{-6}m)}\right) = 22.3°$



$\tan\theta = y/L$ → $y = L\tan\theta = (10m)\tan(22.3°)$

$\boxed{y = 4.11m}$ not a very small angle since y is not $\ll L$

(4 pts, no partial credit) **Problem 8.** This equation:

$$\frac{d^2x}{dt^2} = -\frac{\gamma}{m}\frac{dx}{dt} - \frac{k}{m}x$$

has a solution (meaning a function for which the equation is true) of the form $x(t) = Ae^{-t/\tau}\cos(\omega t)$. That solution represents, for example, a spring with an oscillating mass hanging from it, where the amplitude of oscillation continually decreases due to air resistance. For homework, and for a problem on a previous exam, you represented $x(t)$ as a complex exponential,

$$x(t) = Ae^{t(-\frac{1}{\tau}+i\omega)}$$

took derivatives, and plugged them into the equation. Doing so allowed you to get an equation with both real and imaginary parts to it. Setting the real parts of the left hand side equal to the real parts of the right hand side, and similarly with the imaginary parts, allowed you to find two equations. Those two equations then allowed you to solve for the values of τ and ω (the oscillation frequency and decay time) in terms of the given quantities k, m, and γ. (That's all assuming you did the problem correctly, of course.)

For this problem, go through the same procedure, but stop after you get to the two equations you could use to solve for τ and ω.

$$\frac{dx}{dt} = \left(-\frac{1}{\tau} + i\omega\right) A e^{t(-1/\tau + i\omega)}$$

$$\frac{d^2x}{dt^2} = \left(-\frac{1}{\tau} + i\omega\right)^2 A e^{t(-\frac{1}{\tau} + i\omega)}$$

Plug into equation:

$$\left(-\frac{1}{\tau} + i\omega\right)^2 \cancel{Ae^{t(-\frac{1}{\tau}+i\omega)}} = -\frac{\gamma}{m}\left(-\frac{1}{\tau} + i\omega\right) \cancel{Ae^{t(-\frac{1}{\tau}+i\omega)}} - \frac{k}{m}\cancel{Ae^{t(-\frac{1}{\tau}+i\omega)}}$$

$$+\frac{1}{\tau^2} - \frac{2i\omega}{\tau} - \omega^2 = +\frac{\gamma}{m\tau} - \frac{\gamma i\omega}{m} - \frac{k}{m}$$

Real: $\boxed{+\frac{1}{\tau^2} - \omega^2 = \frac{\gamma}{m\tau} - \frac{k}{m}}$

Imag: $-\frac{2i\omega}{\tau} = -\frac{\gamma i\omega}{m}$

$$\frac{2\cancel{\omega}}{\tau} = +\frac{\gamma\cancel{\omega}}{m}$$

$$\boxed{\frac{2}{\tau} = \frac{\gamma}{m}}$$

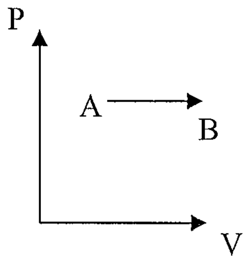
Two equations you get from the real and imaginary parts: (you must get both correct to get any credit for this problem)

(simplified as much as possible) Eqn from real parts: ________________________________

(simplified as much as possible) Eqn from imaginary parts: ________________________________

$C_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R$

(4 pts, no partial credit) **Problem 9**. Heat is added to 2 moles of a diatomic ideal gas at 600K, while keeping its pressure constant as shown in the diagram. Heat comes in from a reservoir kept at 601 K, which cases the temperature of the gas to increase to 601K as shown (in exaggerated form) in the P-V diagram. What was the change in entropy of the universe during this process? (You can assume that the heat lost by the reservoir is equal to the heat added to the gas.)



$\Delta S_{tot} = \Delta S_{gas} + \Delta S_{surroundings}$

$\Delta S = \int \frac{dQ}{T}$ for each part

gas: C_P process

$Q = nC_P\Delta T$

$dQ = nC_P dT$

$\Delta S = \int \frac{nC_P dT}{T}$

$= nC_P \ln T_2/T_1$

$= n\left(\frac{7}{2}R\right)\ln T_2/T_1$

$= 2\left(\frac{7}{2}\cdot 8.31\right)\ln\left(\frac{601}{600}\right)$

$= +.0968693$ J/K

reservoir C_T process

$\Delta S = \int \frac{dQ}{T}$

$= \frac{1}{T}\int dQ$

$= \frac{1}{T}\, Q_{\text{added to reservoir}}$

$= -\frac{1}{T}\, Q_{\text{added to gas}}$

$= -\frac{1}{T}\, nC_P\Delta T$

$= -\frac{1}{601}(2)\left(\frac{7}{2}\cdot 8.31\right)(601-600)$

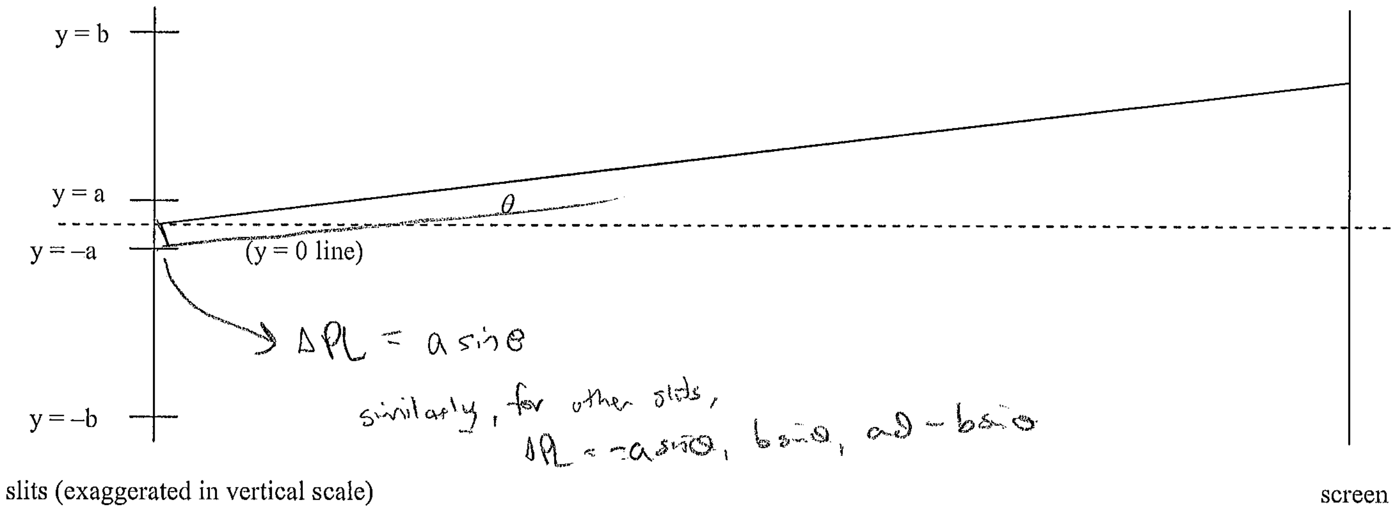
$= -.0967887$ J/K

$\Delta S_{tot} = \quad + $

$= \boxed{8.06\times 10^{-5}\ \text{J/K}}$

Notice even for this tiny temperature change, ΔS_{total} is positive!

(4 pts, no partial credit) **Extra Credit**. Deduce the diffraction pattern (intensity vs angle θ) of four infinitely narrow slits, where the slit separation distances are given in the figure.



$\Delta PL = a\sin\theta$

similarly, for other slits,
$\Delta PL = -a\sin\theta,\ b\sin\theta,\ \text{and } -b\sin\theta$

$$E_{tot} = \sum E_{\text{each slit}}$$

$$= E_0\left(e^{i\phi_1} + e^{i\phi_2} + e^{i\phi_3} + e^{i\phi_4}\right)$$

phase shift for each ΔPL: $\phi = \left(\frac{\Delta PL}{\lambda}\right)2\pi$

$$= E_0\Big[\underbrace{e^{i\frac{a\sin\theta}{\lambda}2\pi} + e^{-i\frac{a\sin\theta}{\lambda}2\pi}}_{2\cos\left(\frac{2\pi a\sin\theta}{\lambda}\right)} + \underbrace{e^{i\frac{b\sin\theta}{\lambda}2\pi} + e^{-i\,b\sin\theta\,2\pi/\lambda}}_{2\cos\left(\frac{2\pi b\sin\theta}{\lambda}\right)}\Big]$$

$$= 2E_0\left[\cos\left(\frac{2\pi a\sin\theta}{\lambda}\right) + \cos\left(\frac{2\pi b\sin\theta}{\lambda}\right)\right]$$

$I \propto |E|^2$

$$\boxed{I = I_0\left[\cos\left(\frac{2\pi a\sin\theta}{\lambda}\right) + \cos\left(\frac{2\pi b\sin\theta}{\lambda}\right)\right]^2}$$