Fall 2010 RED barcode here
Physics 123 section 2
Exam 2
Colton 2-3669 **Please write your CID here _____**

No time limit. One 3×5 note card (handwritten, both sides). No books. Student calculators OK.

Constants which you may or may not need:

$g = 9.8$ m/s^2
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
$k_B = 1.381 \times 10^{-23}$ J/K
$N_A = 6.022 \times 10^{23}$
$R = k_B \cdot N_A = 8.314$ J/mol·K
$\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4
$I_0 = 10^{-12}$ W/m^2
Density of water: 1000 kg/m^3
Density of air: 1.29 kg/m^3

Density of aluminum: 2700 kg/m^3
Linear exp. coeff. of aluminum: 24×10^{-6} /°C
Linear exp. coeff. of copper: 17×10^{-6} /°C
Linear exp. coeff. of steel: 11×10^{-6} /°C
Specific heat of water: 4186 J/kg·°C
Specific heat of ice: 2090 J/kg·°C
Specific heat of steam: 2010 J/kg·°C
Specific heat of aluminum: 900 J/kg·°C
Specific heat of copper: 387 J/kg·ºC

Latent heat of melting (water): 3.33×10^5 J/kg
Latent heat of boiling (water): 2.26×10^6 J/kg
Latent heat of boiling (liquid nitrogen): 1.98×10^5 J/kg
Thermal conduct. of alum.: 238 J/s·m·°C
Thermal conduct. of copper: 397 J/s·m·°C
Speed of sound in air: 343 m/s (unless otherwise specifed)

Conversion factors which may or may not be helpful:

1 inch = 2.54 cm
1 m^3 = 1000 L

1 atm = 1.013×10^5 Pa = 14.7 psi
1 eV = 1.602×10^{-19} J

$$T_F = \frac{9}{5}T_C + 32$$
$$T_K = T_C + 273.15$$

Other equations which you may or may not need to know:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Surface area of sphere = $4\pi r^2$

Volume of sphere = $(4/3)\pi r^3$

Instructions:

- Record your answers to the multiple choice questions ("Problem 1" on the next page) on the bubble sheet.
- To receive full credit on the worked problems, please show all work and write neatly.
- In general, to maximize your partial credit on worked problems you get wrong it's good to solve problems algebraically first, then plug in numbers (with units) to get the final answer. Draw pictures and/or diagrams to help you visualize what the problems is stating and asking, and so that your understanding of the problem will be clear to the grader.
- Unless otherwise instructed, give all numerical answers for the worked problems in SI units, to 3 or 4 significant digits. For answers that rely on intermediate results, remember to keep extra digits in the intermediate results, otherwise your final answer may be off. Be especially careful when subtracting two similar numbers.
- Unless otherwise specified, treat all systems as being frictionless (e.g. fluids have no viscosity).

Scores: (for grader to fill in)

Problem 1	_____	Problem 6	_____
Problem 2	_____	Problem 7	_____
Problem 3	_____	Problem 8	_____
Problem 4	_____	Problem 9	_____
Problem 5	_____	Extra Credit	_____
		Total	_____

Did you remember to write down your CID at the top of the page? _____

(15 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer. **Fill in your answers on the bubble sheet.**

1.1. You have two balloons, one filled with air and one filled with helium. If you put both balloons into a tub of liquid nitrogen, which one will end up with the *smallest* volume?
   a. the air balloon
   b. the helium balloon
   c. they will end up with the same volume

1.2. The second law of thermodynamics is a statement of:
   a. conservation of energy
   b. conservation of (regular) momentum
   c. conservation of angular momentum
   d. conservation of mass/volume
   e. probability

1.3. As I'm taking data in my lab, I often take data from a light detector and average the data for 1 second for each data point. You can assume that this represents data from 1,000,000 photons, for example. Suppose I decide to average the data for 2 seconds per point instead (collecting 2,000,000 photons, for example). How much better is my signal-to-noise ratio likely to be? Hint: Think about the statistical fluctuations (the "noise") likely to occur in each case.
   a. the same
   b. $\sqrt{2}$ times better
   c. 2 times better
   d. 4 times better
   e. 8 times better

1.4. In the "ladies belt demo" (the belt was like a "closed-closed" string), suppose the fundamental frequency is seen at 500 Hz. What frequency will have five antinodes?
   a. 800 Hz
   b. 1250
   c. 1500
   d. 2000
   e. 2500
   f. 3000 Hz

1.5. When a "wave" is actually a finite duration pulse in a medium with dispersion, the group velocity is the speed at which:
   a. the overall pulse envelope propagates
   b. all individual frequency components propagate
   c. the average frequency component propagates
   d. none of the above
   e. more than one of the above

1.6. You should have found the wave speed of waves on your slinky to be directly proportional to the slinky's length. (This was true for both longitudinal and transverse waves.) Compare the time it takes waves to do a round-trip path when the slinky is stretched to 5 feet compared to when it is stretched to 10 feet.
   a. $t_{5ft} > t_{10ft}$
   b. $t_{5ft} \approx t_{10ft}$
   c. $t_{5ft} < t_{10ft}$

1.7. Complete the following based on our in-class discussions: To localize a wave in space, you need lots of _____
   a. amplitude
   b. energy
   c. patience
   d. phase
   e. wavenumbers

1.8. The sound of a trumpet playing a note is qualitatively different than the sound of a flute playing the same note. Why is that?
   a. The two notes have different amplitudes.
   b. The two notes have different durations.
   c. The two notes have different fundamental frequencies.
   d. The two notes have different phases.
   e. The two notes have different strengths of harmonics.

1.9. The wavefunction for a particular wave on a string is given by: $f(x,t) = 4\cos(2x - 6t + 5)$. You may assume that the numbers in that equation and in the answer choices below are all given in terms of the appropriate SI units. What is the amplitude?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.10. Same equation. What is the wavelength?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.11. Same equation. What is the wavenumber (k)?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.12. Same equation. What is the period?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.13. Same equation. What is the angular frequency (ω)?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.14. Same equation. What is the phase (ϕ, in radians)?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

1.15. Same equation. What is the wave's speed?

a. 2
b. 3
c. 4
d. 5
e. 6
f. $2\pi/2$
g. $2\pi/3$
h. $2\pi/4$
i. $2\pi/5$
j. $2\pi/6$

(11 pts) **Problem 2**. (a) A car (traveling at 44 m/s) is chasing you on your bike (traveling at 12 m/s). The car driver honks her horn and emits a tone of 500 Hz. Use 343 m/s for the speed of sound. What frequency do you hear?

(b) The animated gif I showed in class where the group and phase velocities were in opposite directions had the following dispersion relation: $\omega = -1/k^2$. (You can assume that the number "1" in the equation has the appropriate units to make ω be in rad/s and k in rad/m.) It was made up of 21 frequency components centered around $k = 1$ rad/m. Find the group velocity of the pulse and the phase velocity of the central phase.

v_{group} = _____

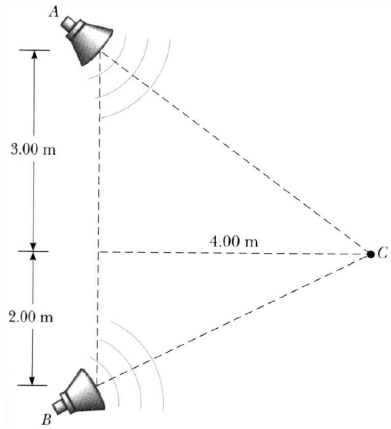
v_{phase} = _____

(10 pts) **Problem 3**. Guitar players often tune their instrument using harmonics. Imagine that I use an electronic tuner to tune my low E string to 164.814 Hz, precisely the correct frequency for an equal temperament scale referenced to the standard frequency of 440.000 Hz for the A above middle C. Now I tune my next string, the A string—which is 5 half-steps higher in frequency, a musical "fourth"— using harmonics. I do this by lightly touching the E string 1/4 of the way from the end of the string and lightly touching the A string 1/3 of the way from the end of the string and adjusting its frequency so that the beats go away.

(a) What is the frequency ratio of the two notes and why does this tuning method work?

(b) What is the difference in frequency between an A tuned that way and an A tuned according to the equal temperament scale?

(11 pts) **Problem 4**. Two small speakers emit spherical waves, both with outputs of 2 mW of sound power.



(a) Assuming the two waves are incoherent so that the *intensities* add, determine the intensity (W/m^2) and sound level (dB) experienced by an observer located at point C.

intensity = _____ W/m^2

sound level = _____ dB

(b) Assuming the two waves are coherent, so that the *amplitudes* add (to produce a pattern of minima and maxima), what is the lowest frequency that will produce a maximum at point C? The two speakers are governed by sinusoidal voltage sources that have the same frequency and are in phase with each other. Use v_{sound} = 343 m/s.

(12 pts) **Problem 5**. Add these two cosine functions together using complex number techniques to determine the amplitude and phase of the resulting function: $f_1 = 3\cos(4t + 5)$, $f_2 = 4\cos(4t + 6)$, and thereby write down the resulting function. The numbers "4" and "6" have units of rad/s and radians, respectively. Hint: be careful as to whether your calculator is in degrees or in radians mode. (If you know the method, you don't necessarily have to write down any complex numbers to solve this problem.)

Amplitude = _____

Phase = _____ radians, = _____ degrees

Resulting function $f(t) = f_1(t) + f_2(t)$ = _____

(8 pts) **Problem 6**. A sound wave travels from air (v = 343 m/s) into a small lake (v = 1493 m/s), coming from a source far from the lake, and directly above it, so that the wave crests are essentially parallel to the water as it enters. This is thus a 1D situation, where the same equations we derived for waves on a string are applicable.

(a) What fraction of the incident wave's amplitude is reflected?

(b) What fraction of the incident wave's amplitude is transmitted? Hint: don't be surprised if this result…well, surprises you.

(c) What fraction of the incident wave's power is reflected?

(d) What fraction of the incident wave's power is transmitted?

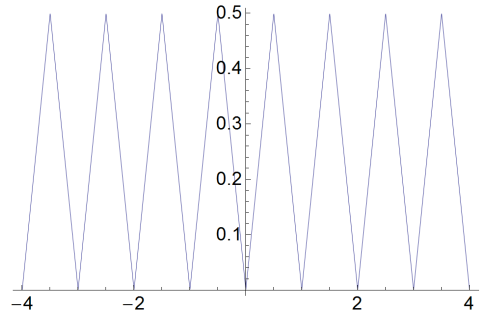
(12 pts) **Problem 7**. This equation:

$$\frac{d^2x}{dt^2} = -\frac{\gamma}{m}\frac{dx}{dt} - \frac{k}{m}x$$

has a solution (meaning a function for which the equation is true) of the form $x(t) = Ae^{-t/\tau}\cos(\omega t)$. That solution represents, for example, a spring with an oscillating mass hanging from it, where the amplitude of oscillation continually decreases due to air resistance.

By representing $x(t)$ as a complex exponential, $x(t) = Ae^{t(-\frac{1}{\tau}+i\omega)}$ and plugging it into the equation, find what τ and ω must be in terms of k, m, and γ, to make the equation true. (Yes, this is the same as your homework problem HW 16-5. Demonstrate that you can do the problem, don't just quote the answer to that problem.)

(15 pts) **Problem 8**. In my lab I have a voltage "function generator" which can produce triangular-shaped voltage patterns such as the one shown. The x-axis is seconds; the y-axis is volts. (You can ignore units for the rest of this problem.) From 0 to 0.5 s it sends out a signal that is $f(x) = x$. From 0.5 s to 1.0 s it sends out a signal that is $f(x) = 1 - x$. Then the signal repeats.



Determine the Fourier coefficients of this function and write $f(x)$ as a sum of sines, cosines, and a constant term, as applicable. If any of the Fourier coefficients have obvious values, state your reasons why it/they are obvious.

Potentially useful integrals:

$$\int \sin(2\pi nx)\,dx = -\frac{\cos(2\pi nx)}{2\pi n}$$

$$\int \cos(2\pi nx)\,dx = \frac{\sin(2\pi nx)}{2\pi n}$$

$$\int x\sin(2\pi nx)\,dx = -\frac{x\cos(2\pi nx)}{2\pi n} + \frac{\sin(2\pi nx)}{4\pi^2 n^2}$$

$$\int x\cos(2\pi nx)\,dx = \frac{\cos(2\pi nx)}{4\pi^2 n^2} + \frac{x\sin(2\pi nx)}{2\pi n}$$

Note: although you should recognize that sin(nπ) and sin(2nπ) = 0 for all integer values of n, you can leave your answers in terms of cos(nπ) and cos(2nπ); that is, you don't have to work out the pattern of your answers in terms of odds/evens or (-1) to some power.

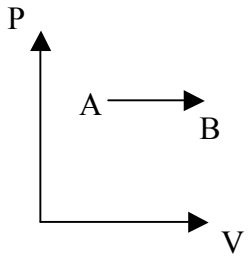
a_0 (constant term) = _____

a_n (cosine coefficients) = _____

b_n (sine coefficients) = _____

f(x) = _____
(function written out as an infinite series)

(6 pts, no partial credit) **Problem 9**. Heat is added to 4 moles of a monatomic ideal gas at 300K, while keeping its pressure constant as shown in the diagram. Heat comes in from a reservoir kept at 710 K, which cases the temperature of the gas to increase to 700K as shown in the P-V diagram. What was the change in entropy of the universe during this process? (Hint: find the change in entropy for the gas and for the reservoir separately, then add them together. By the 2nd Law, your final answer must be positive. You can assume that the heat lost by the reservoir is equal to the heat added to the gas.)



(4 pts, no partial credit) **Extra Credit**.

Suppose you attach a rope (mass m, length L) to the ceiling and let it dangle down so that it just barely reaches the floor without touching. How long will it take for a transverse pulse to travel from the bottom of the rope to the top? Put your answer in terms of m, g, and L.

Hint: You can do this in two steps. First, pick a point on the rope a distance x up from the bottom of the rope, and draw a free-body diagram for that point. There's some weight (but not all the weight) pulling down, and some tension pulling up. That should give you tension as a function of distance. You may assume that the rope's linear mass density will not vary with height. You already know how the wave speed depends on tension and linear mass density, so you should then be able to figure out the wave speed as a function of distance. Then, use that information and some calculus to figure out the answer to the problem.